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VIOLATION OF THE $\Delta I = \frac{1}{2}$ RULE IN $D \rightarrow \pi\pi$ DECAYSFrancesco Sannino^(a,b)

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ABSTRACT

A strong violation of the $\Delta I = \frac{1}{2}$ rule has experimentally been found in the $D \rightarrow \pi\pi$ decays [1]. In this letter we will show that the order of magnitude of this violation can be understood in terms of the pure quantum chromo dynamics corrections to the weak interactions.

• Introduction

The approximate $\Delta I = \frac{1}{2}$ rule was firstly established in the kaon decays and until now never experimentally violated.

The strong $\Delta I = \frac{3}{2}$ suppression is still now not completely understood and so it remains one of the more interesting and open problems in elementary particle physics. It was suggested [2] that one possible solution to this puzzle could be the coupling of the strong interactions to the weak currents. This idea for the kaon decays cannot quantitatively work as QCD under 1 *GeV* is not perturbative and relatively big soft gluon corrections should be expected. Nevertheless it gives us a non trivial physical picture in order to understand the phenomenological rule. The suppression in this scheme is only due to the underlying color dynamics of the interacting gluons and quarks. Now if we assume that the gluon exchanges (hard and soft) really affect the weak currents, we expect a *dynamical* correction, in other words, the corrections should depend on the energy scale involved in the decays. It is really the different energy scale between the *D* and *K* decays which must play a fundamental role in the understanding of the $\Delta I = \frac{1}{2}$ violation in *D* decays. It is also possible to give a *naive* physical picture in which as the *c* mass is greater than the *s* mass we expect that the dynamical gluon corrections don't strongly affect the weak operators for the *D* decays. In this letter we haven't used the standard factorization approach which is based on the valence-quark assumption and vacuum-insertion approximation. In that scheme the matrix elements of two quark bilinear operators are saturated by

the vacuum intermediate states in all possible ways. We believe that even if the factorization scheme has some suggestive features it actually breaks the fundamental symmetries of the whole effective weak operators, leading to some wrong results [11,14]. On the contrary, we assume that the total symmetries of the QCD regularized bilinear weak operators are preserved by the hadronization and this in turns allows us to easily relate the $D \rightarrow \pi\pi$ weak amplitudes to each other.

• **Experimental starting point**

The usual isospin decomposition for the $D \rightarrow \pi\pi$ decays can be written as in the kaon decays [1,4,5]

$$\begin{aligned}
 A^{+0} &= \sqrt{\frac{3}{2}} A_2 e^{i\delta_2} \\
 A^{+-} &= \frac{1}{\sqrt{3}} A_2 e^{i\delta_2} + \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} \\
 A^{00} &= \sqrt{\frac{2}{3}} A_2 e^{i\delta_2} - \frac{1}{\sqrt{3}} A_0 e^{i\delta_0}
 \end{aligned} \tag{1.1}$$

where A^{+0} , A^{+-} and A^{00} are the amplitudes for $D^+ \rightarrow \pi^+\pi^0$, $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow \pi^0\pi^0$ respectively.

In this standard notation the δ_2 and δ_0 phases are related to the strong final state interactions of $\pi\pi$ elastic scattering. This decomposition, as well known, is based on the Watson's theorem [5,6] which uses the unitarity condition and the elastic structure of the strong interaction for the *in* and *out* states. If we relax the condition for the elastic structure of the final state interactions, the decomposition (1.1) can still be made, but δ_2 and δ_0 should just be interpreted as new parameters. Some author [11] uses complex phases to parametrize a possible strong $\pi\pi$ inelastic final state interaction. Here we will assume elastic $\pi\pi$ final state interactions. The experimental analysis based on (1.1) leads to the following very interesting results [1]:

$$\left| \frac{A_2}{A_0} \right| = 0.72 \pm 0.13 \pm 0.11 \quad (1.2)$$

and

$$\cos(\delta_2 - \delta_0) = 0.14 \pm 0.13 \pm 0.09 \quad (1.3)$$

As the A_2 amplitude is comparable to the A_0 amplitude, the $\Delta I = \frac{1}{2}$ rule, established in the $K \rightarrow \pi\pi$ decays, is badly broken here. Now we are going to understand this different behaviour in terms of the underlying quantum chromo dynamics.

• **Dynamical behaviour of the $\Delta I = \frac{1}{2}$ rule**

The bare weak non-leptonic hamiltonian which contributes to $D \rightarrow \pi\pi$ is [3,10]:

$$H_W = \frac{G_F}{\sqrt{2}} \sin(\vartheta_c) \cos(\vartheta_c) \left[(\bar{d}u)(\bar{c}d) + (\bar{u}d)(\bar{d}c) \right] \quad (2.1)$$

where $(\bar{q}_2 q_1)$ denotes an uncolored left $(V - A)$ current and ϑ_c is the Cabibbo angle. Taking into account hard gluon exchange we have the effective non leptonic weak hamiltonian [2,3,10]:

$$H_W^{eff.} = \frac{G_F}{2\sqrt{2}} \sin(\vartheta_c) \cos(\vartheta_c) [a_+ I_+ + a_- I_-] + h.c. \quad (2.2)$$

where

$$I_{\pm} = (\bar{u}d)(\bar{d}c) \pm (\bar{d}d)(\bar{u}c) \quad (2.3)$$

a_+ and a_- are coefficients which summarize the underlying dynamics of the hard gluon-quark interactions. Their explicit expression is

$$a_{\pm} = \left(\frac{\alpha_S(m_c)}{\alpha_S(M_W)} \right)^{\gamma_{\pm}} \quad (2.4)$$

where

$$\gamma_- = -2\gamma_+ = \frac{4}{b} \quad b = 11 - \frac{2}{3}N_f \quad N_f = 4 \quad (2.5)$$

and γ_{\pm} are called the anomalous dimensions. The energy scale dependence of the strong coupling constant is

$$\alpha_S(Q^2) = \frac{4\pi}{b \ln\left(\frac{Q^2}{\Lambda^2}\right)} \quad (2.6)$$

The standard notation has been used here. We use $\alpha_S(M_W) \cong 0.113$ [17], while for $\alpha_S(m_c)$, due to the big uncertainty on Λ , we use two different values: $\alpha_S(m_c) \cong 0.2, 0.3$. It is easy to show that

$$a_+ \cong 0.87 \qquad a_- \cong 1.31 \quad (2.7)$$

$$a_+ \cong 0.79 \qquad a_- \cong 1.60$$

where the first and second line are respectively due to $\alpha_S(m_c) \cong 0.2$ and $\alpha_S(m_c) \cong 0.3$. The new effective operators I_{\pm} have well defined behaviours under isospin transformations. Indeed, we will see that I_- is a pure $\Delta I = \frac{1}{2}$ operator while I_+ is a mixture of $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{1}{2}$ [5]. Looking at the a_{\pm} coefficients we observe a relatively small dynamical suppression of the $\Delta I = \frac{3}{2}$ operator so that we expect a failure of the $\Delta I = \frac{1}{2}$ rule in the D decays. The crucial point is that at the m_c scale this perturbative picture should work better than at the kaon scale as we will show. We will not consider the penguin diagrams, which are fundamental in the kaon decays, because due to SU(3) flavour symmetry they shouldn't play any fundamental role in the D decays [5,10].

• Theoretical Results and Conclusions

To make quantitative estimates we will use the isospin properties of the I_{\pm} operators and assume that the subsequent hadronization actually preserves the whole non-dynamical information carried by the effective weak operators. Let us expand the I_{\pm} operators in terms of operators with definite isospin:

$$I_+ = \frac{2}{\sqrt{3}} T_{\frac{1}{2}}^{\frac{3}{2}} + \frac{1}{\sqrt{6}} \hat{T}_{\frac{1}{2}}^{\frac{1}{2}} - \frac{1}{\sqrt{2}} T_{\frac{1}{2}}^{\frac{1}{2}} \quad (3.1)$$

$$I_- = \sqrt{\frac{3}{2}} \hat{T}_{\frac{1}{2}}^{\frac{1}{2}} + \frac{1}{\sqrt{2}} T_{\frac{1}{2}}^{\frac{1}{2}} \quad (3.2)$$

As claimed I_- is a pure $\Delta I = \frac{1}{2}$ operator while I_+ is a combination of $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{1}{2}$. The two different $\Delta I = \frac{1}{2}$ operators $T_{\frac{1}{2}}^{\frac{1}{2}}$ and $\hat{T}_{\frac{1}{2}}^{\frac{1}{2}}$ are due respectively to the following products of two SU(2) isospin representations $0 \otimes \frac{1}{2}$ and $1 \otimes \frac{1}{2}$. Now using the Bose symmetrized $\pi\pi$ final states and the Wigner-Eckart theorem we will be able to express the ratio $\left| \frac{A_2}{A_0} \right|$ as a function of the dynamical coefficients a_{\pm} and of the reduced matrix elements. The three independent reduced matrix elements are defined as:

$$\begin{aligned} M_2 &:= \langle 2 \| T^{\frac{3}{2}} \| 1/2 \rangle \\ M_0 &:= \langle 0 \| T^{\frac{1}{2}} \| 1/2 \rangle \\ \hat{M}_0 &:= \langle 0 \| \hat{T}^{\frac{1}{2}} \| 1/2 \rangle \end{aligned} \quad (3.3)$$

It's finally possible to obtain the following model independent expression

$$\left| \frac{A_2}{A_0} \right| = 2a_+ \sqrt{2} \left| \frac{M_2}{\hat{M}_0} \right| \left| (a_+ + 3a_-) + \sqrt{3} \frac{M_0}{\hat{M}_0} (a_- - a_+) \right|^{-1} \quad (3.4)$$

To obtain quantitative results we need a scheme to deduce the three reduced matrix elements M_2 , M_0 and \hat{M}_0 . Unfortunately, we still do not have a reliable model for the non leptonic matrix elements and that is why after almost 30 years the $\Delta I = \frac{3}{2}$ suppression in $K \rightarrow 2\pi$ is still unsatisfactorily explained. On the contrary, we can see that, due to different energy scales, the simplest ansatz on those reduced matrix elements is able to explain the new experimental data for $D \rightarrow 2\pi$. Our hypothesis for the reduced matrix elements is that the non perturbative strong interactions which are involved in the subsequent hadronization do not break the total effective operators symmetries. So we simply assume:

$$M_2 = M_0 = \hat{M}_0 \quad (3.5)$$

We can than rewrite (3.4) in terms only of the a_{\pm}

$$\left| \frac{A_2}{A_0} \right| = \frac{2a_+ \sqrt{2}}{\left| (a_+ + 3a_-) + \sqrt{3}(a_- - a_+) \right|} \quad (3.6)$$

Using the numerical expressions for a_{\pm} we have for $\alpha_S(m_c) \cong 0.3$ and $\alpha_S(m_c) \cong 0.2$ respectively:

$$\left| \frac{A_2}{A_0} \right| \cong 0.32, \quad 0.44 \quad (3.8)$$

We can see comparing the theoretical results with the experimental one, which is $\left| \frac{A_2}{A_0} \right| = 0.72 \pm 0.13 \pm 0.11$, that this simple model gives the right order of magnitude and that the theoretical value is about two standard deviations from the experimental one. We also note that experimentally the $\Delta I = \frac{3}{2}$ is more enhanced than the theoretical prediction and that in turn should reflect on the reduced matrix elements. On the contrary if we use the same idea for the K decays, we recall that a too large theoretical $\Delta I = \frac{3}{2}$ is expected [2]. One possible explanation of this interesting puzzle could be a little inelastic $\pi\pi$ scattering in the D decays. We can also have soft-gluon effects, and these are expected, roughly speaking, at the D scale to be of order $\frac{\Lambda_{QCD}}{m_c} \cong 14\%$ with $\Lambda_{QCD} \cong 200 \text{ MeV}$ while we need a correction about 40% to obtain the central experimental value. Another little contribution could come from non-expected penguin operators which should enhance the $\Delta I = \frac{1}{2}$ amplitude.

However, to really disentangle this problem we need new more precise measurements of the ratio $\left| \frac{A_2}{A_0} \right|$. We can conclude observing that our theoretical result indicates that the observed violation of the $\Delta I = \frac{1}{2}$ rule for $D \rightarrow \pi\pi$ is effectively due to the perturbative underlying chromodynamics, which can be identified with the dynamical coefficients a_{\pm} .

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